Novel phase structure for Lattice flavored chemical potential

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Based on JHEP 08(2012)068 [arXiv:1206.0969]

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Chemical potential on the lattice

(1) Continuum-like : $\mu\bar{\psi}_n\gamma_4\psi_n \;\sim\; \mu\psi_n^\dagger\psi_n$

→ quadratic divergence of energy density : $\mathcal{E} \sim \mu^2/a^2$

→ counter terms required *useless....*

(2) Photon-field-like : $\bar{\psi}_n(e^{\mu a}\psi_{n+4} - e^{-\mu a}\psi_{n-4})$ [Hasenfratz-Karsch '83]

→ Abelian gauge invariance kept

→ correct finite energy density : $\mathcal{E} \sim \mu^4$ *useful !*

1.Reconsider (1)*, especially for i*μ.
2.*Utilize* (1) *for another purpose : control #flavors.*
3.Study chiral phase diagram.

1. Lattice fermions

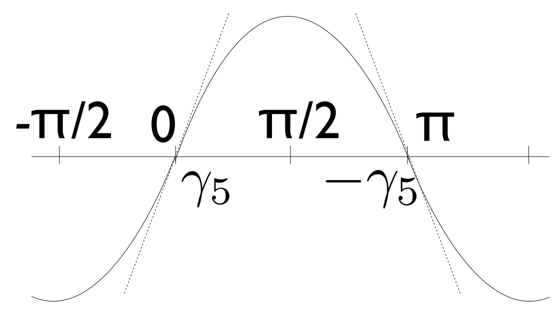
- Doubling problem : Naive chiral&local fermion → 16 species

$$S_{\mathrm{N}} = \sum_n \left[\frac{a^3}{2} \bar{\psi}_n \gamma_\mu (U_{n,\mu}\psi_{n+\mu} - U^\dagger_{n-\mu,\mu}\psi_{n-\mu}) + a^4 m \bar{\psi}_n \psi_n \right]$$

Free propagator

$$D^{-1}(pa) = \frac{-i\gamma_\mu \sin ap_\mu + am}{\sin^2 ap_\mu + a^2 m^2} \quad \rightarrow \quad \frac{1}{a} \underline{\sum_{p_\mu = 0,\pi/a}} \frac{-i(-1)^{\delta_\mu}\gamma_\mu \hat{p}_\mu + m}{\hat{p}^2_\mu + m^2}$$

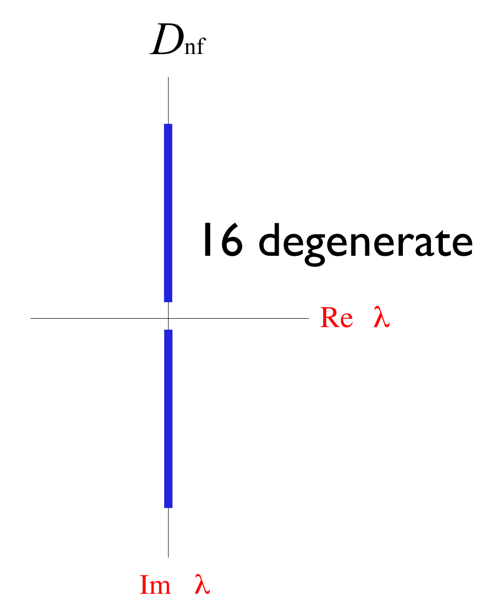
2 poles per dim. → 16 doublers in 4d



Nielsen-Ninomiya

→ *Chiral symmetry v.s. desirable flavor number*

	flavors	chiral	tuning	artifact
Wilson:	1	0	severe	O(a)
Staggered:	4	1	N/A	O(a^2)
Domain-wall	1	1	easy	O(a^2)
Overlap	1	1	N/A	O(a^2)



Dirac spectrum

◆Wilson fermion : species-splitting by mass

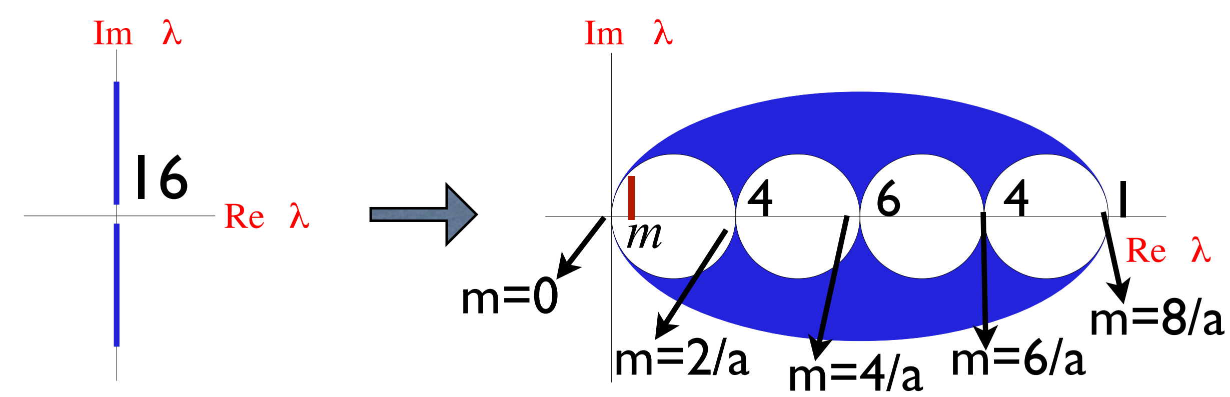
$$+ \quad S_W = \frac{a^5}{2}\bar{\psi}_n(2\psi_n - \psi_{n+\mu} - \psi_{n-\mu})$$

$$\Rightarrow \quad D_W(p) = \frac{1}{a}\sum_{\mu}[i\gamma_\mu \sin ap_\mu + \underline{(1-\cos ap_\mu)}]$$ Flavored mass

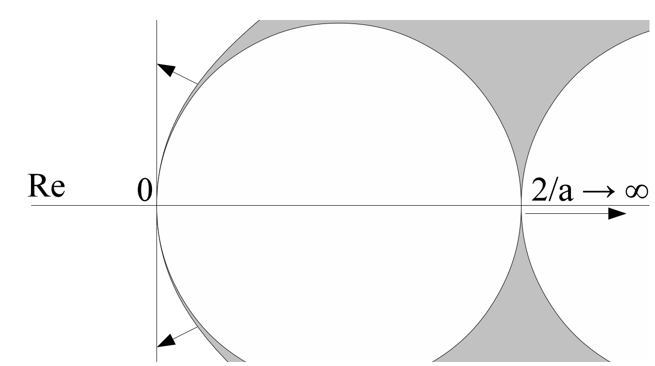
Physical (0,0,0,0) : $D_W(p) = i\gamma_\mu p_\mu + O(a)$

Doubler(π/*a*,0,0,0) : $D_W(p) = i\gamma_\mu p_\mu + \frac{2}{a} + O(a)$

Only one flavor is massless, while others have 1/a mass.



- ◆ 15 species are decoupled → doubler-less
- ◆ 1/a additive mass renormalization → Fine-tune



Species-splitting without breaking chiral symmetry ?

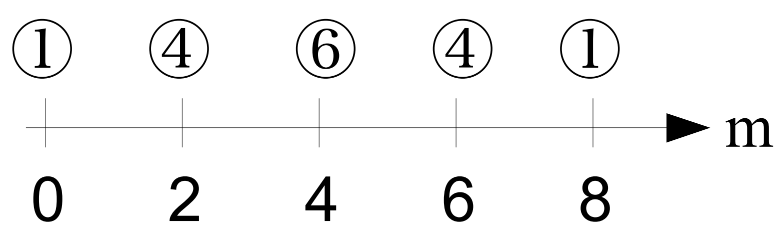
Chiral-symmetric way of lifting species degeneracy

$\bar{\psi}_n \Delta \psi_n$: lifted by flavored-mass

$(i)\bar{\psi}_n \gamma_4 \Delta \psi_n$: lifted by flavored-chemical potential

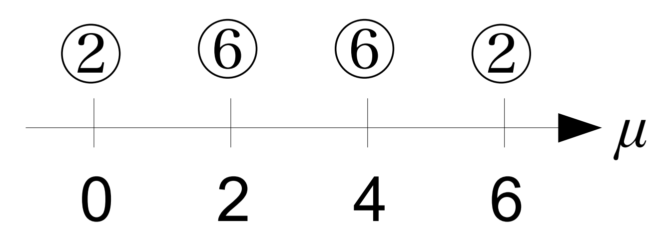
holding chiral symmetry!

Wilson



$$\sum_{\mu}(1-\cos p_\mu)$$

Flavored chemical-pot.



species feel different chemical potential

$$(\mathrm{i})\,\gamma_4 \sum_{j=1}^{3}(1-\cos p_j)$$

Imaginary one is preferred to avoid sign problem.

2. Minimal-doubling

[Karsten '81][Wilczek '87]
[Creutz '07][Borici '07]
[Creutz&Misumi '10]

- #species = 2
- One exact chiral symmetry
- Ultra-Locality

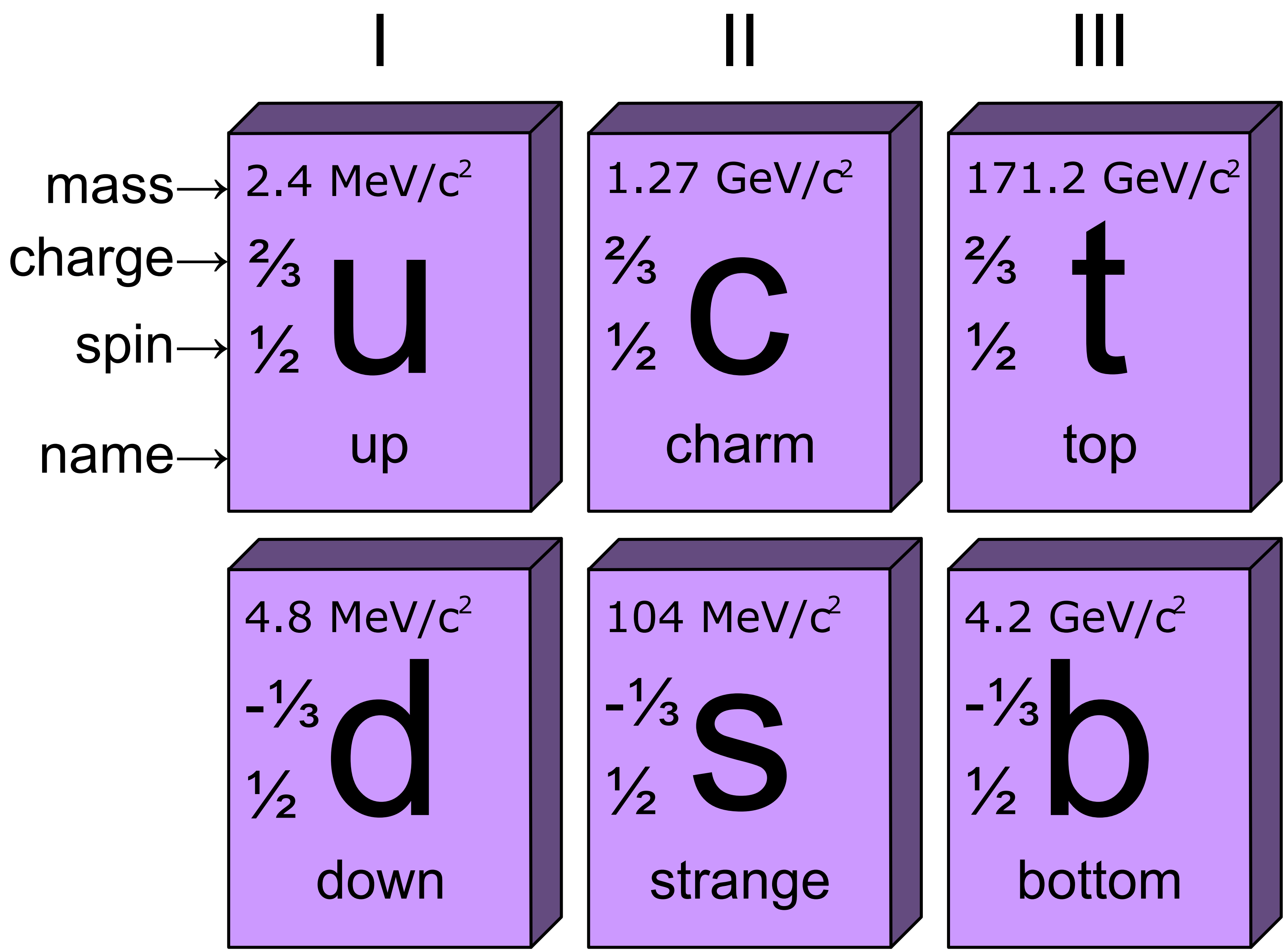
$$D_{\mathrm{KW}}(p) = i\gamma_\mu \sin p_\mu + ir\gamma_4 \sum_{j=1}^{3}(1-\cos p_j) + \underbrace{i\mu_3\gamma_4}_{\text{counterterm}}$$

Wilson-like: not mass, but img chemical potential

cf.) Wilson

$$D_W(p) = i\gamma_\mu \sin p_\mu + r\sum_{\mu=1}^{4}(1-\cos p_\mu) + m$$

Quarks



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2.4 MeV/c²
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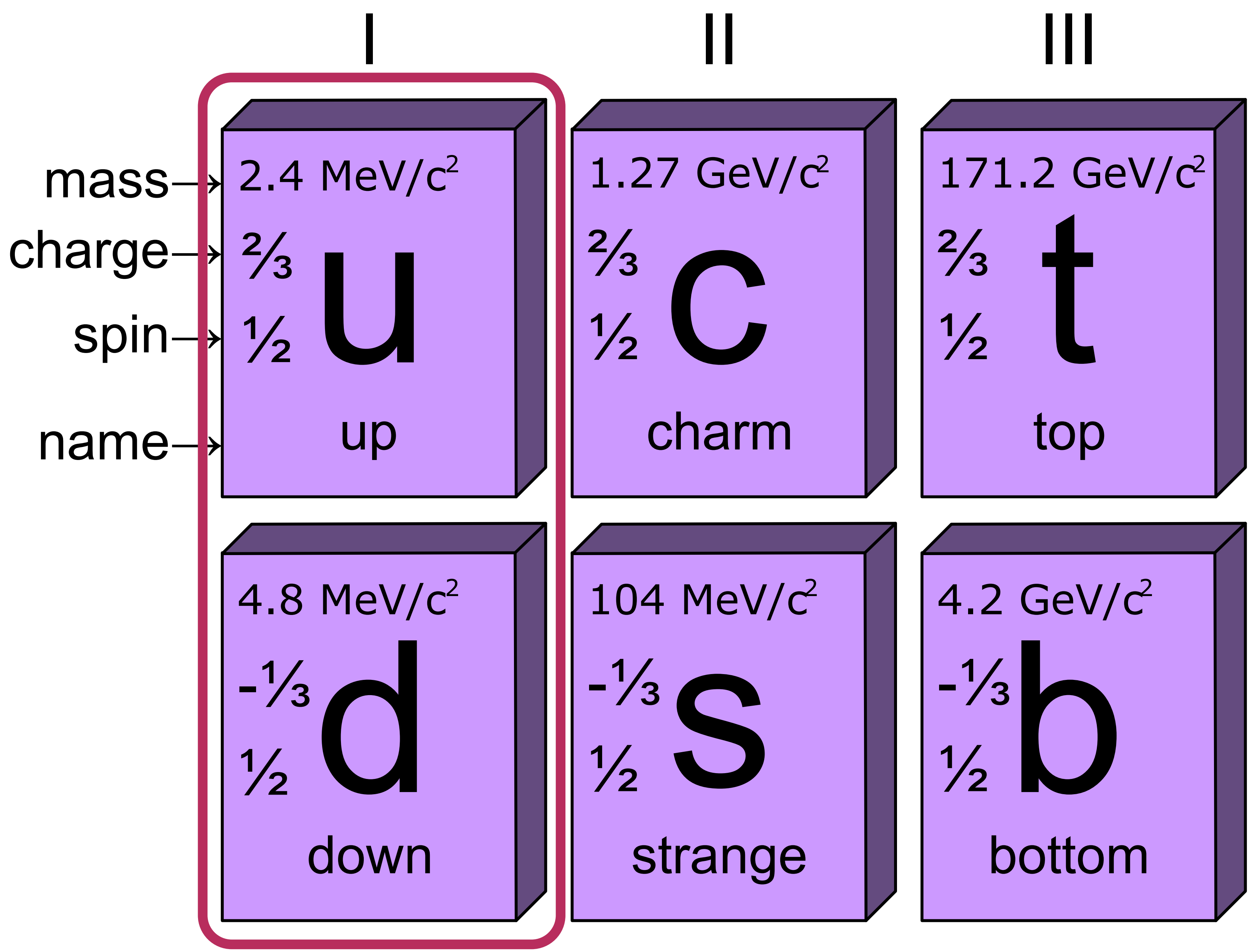
Quarks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-⅓
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Lattice fermions

	#doublers	chiral	spinor
naive	16	exact	4
Wilson	1	none	4
staggered	4	exact	1
min double	2	exact	4

◆MD symmetries

[Bedaque, Buchoff, Tiburzi, Walker-Loud, '08]

1. U(1) chiral
2. P
3. CT
4. Cubic

In a continuum limit ⇒

1. SU(2) chiral
2. P
3. CT
4. Spatial rotation

→ Symmetries of finite-density systems

cf.) Naive fermion with μ

Minimal-doubling → Same symmetry ← Finite density (2flavor)

Same symmetry → Same universality class in cont lim

◆Additive chemical-pot renormalization

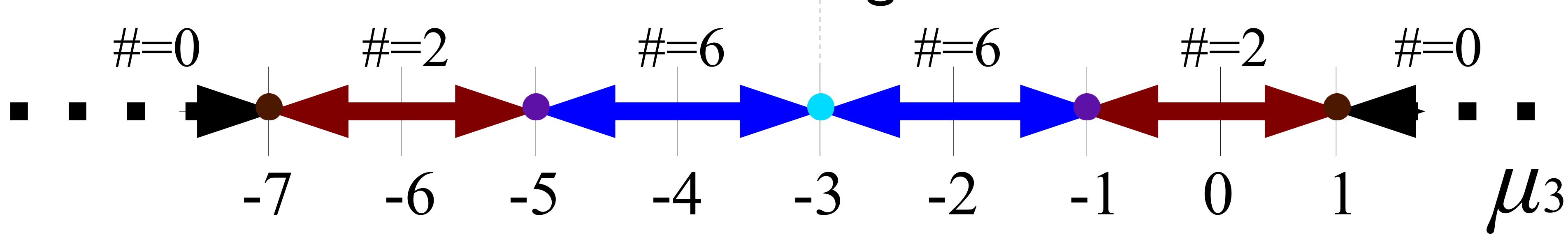
Flavored mass in Wilson → 1/a additive mass ren.

↕

Flavored μ in MD → 1/a additive μ ren.

⇨ To control chem pot, we need to tune $\mu_3 \bar{\psi}_n i\gamma_4 \psi_n$

This ren. can also change # of flavors !



cf.) For (T=0, μ=0) lattice QCD

3 counterterms for a Lorentz-sym cont. limit [Capitani-Creutz-Weber-Wittig '09]

dim3 $\mu_3 \bar{\psi}_n i\gamma_4 \psi_n$ *dim4* $\bar{\psi}_n \gamma_4 D \psi_n$ $F_{i4}F_{i4}$

◆Application to (T,μ) lattice QCD

2-flavor finite-density chiral for Post Sign problem

cf.) Rooting fails for $\mu \neq 0$. High cost for overlap.

Option1: Flavored img μ + photon-like μ

To decouple 14 doublers

physical chem-pot for 2 flavors

⇨ correct energy density

$$\mathcal{E} \equiv \mathcal{I}(\mu, r, \mu_3) - \mathcal{I}(0, r, \mu_3) \sim \mu^4$$

Option2: Flavored real μ + μ_3 fine-tuning

To decouple 14 doublers

physical chem-pot for 2 flavors

Wilson and Minimal-doubling

◆Wilson

- Flavored mass → finite-mass system
- Chiral symmetry breaking
- Additive mass renormalization
- Mass tuning

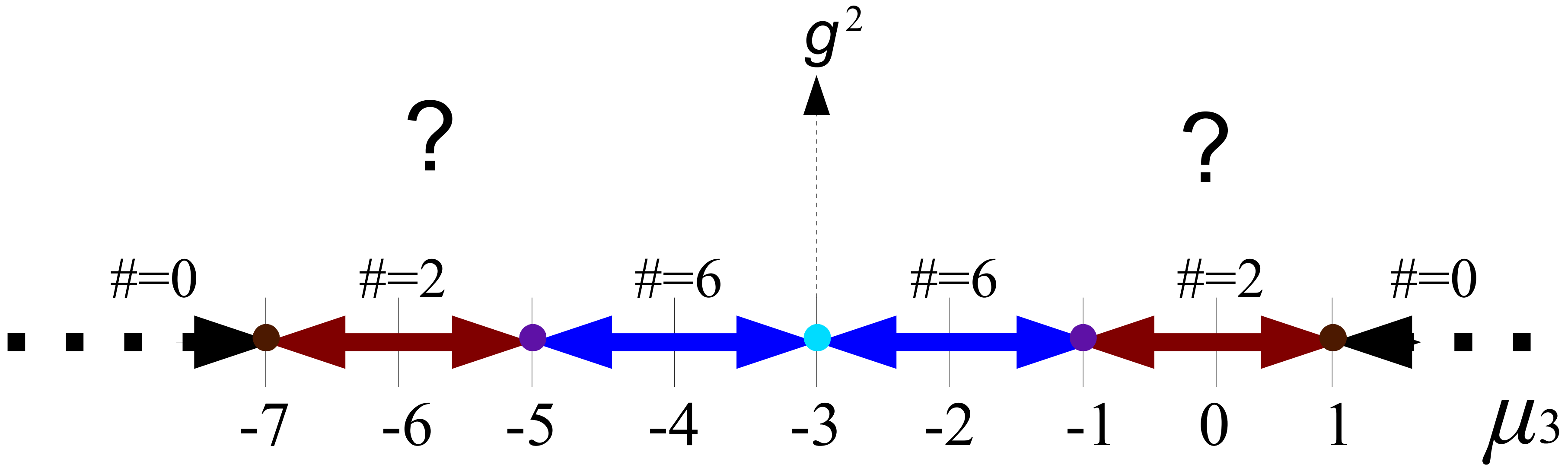
◆MD

- Flavored chemical potential → finite-density
- Spacetime symmetry breaking
- Additive chemical potential renorm.
- μ_3 tuning

3. Phase structure in μ_3-g space

◆Why we need to study

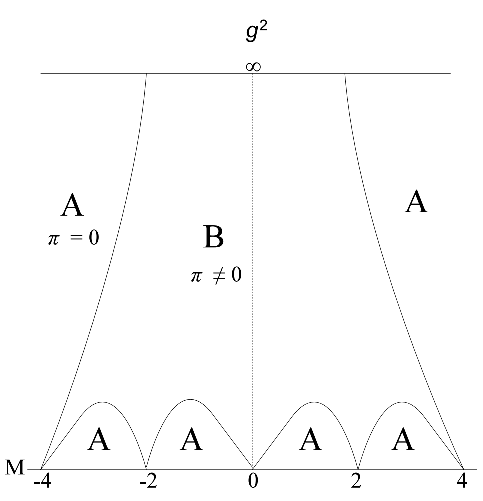
Additive renorm. can change 2-flavor range.



Phase diagram can give guiding principle for μ_3 tuning.

cf.) Aoki phase in Wilson

Chiral limit is taken along with the phase boundary.



It is essential both for zero & finite-(T,μ).

i) Strong-coupling limit

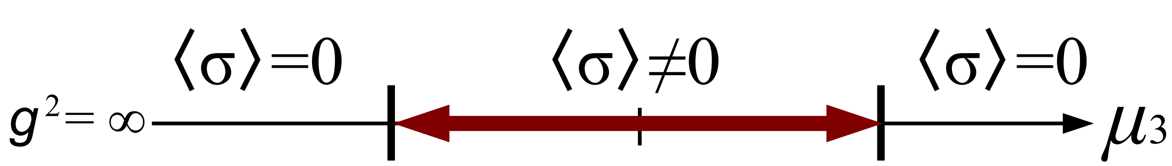
1. Link variable integral
2. Bosonization → meson potential
3. Determine the vacuum

- SC meson potential for MD

$$S_{\text{eff}} = -4N_c \text{Vol.} \mathcal{V}_{\text{eff}}(\sigma, \pi_4)\,,$$

$$\mathcal{V}_{\text{eff}}(\sigma, \pi_4) = \frac{1}{2}\log(\sigma^2 + \pi_4^2) - m\sigma + (\mu_3 + 3r)\pi_4$$
$$- \frac{1}{4}\left[3(1+r^2) + (1+d_4)^2\right]\sigma^2 - \frac{1}{4}\left[3(1-r^2) - (1+d_4)^2\right]\pi_4^2\,.$$

→ *Non-trivial chiral phase structure*

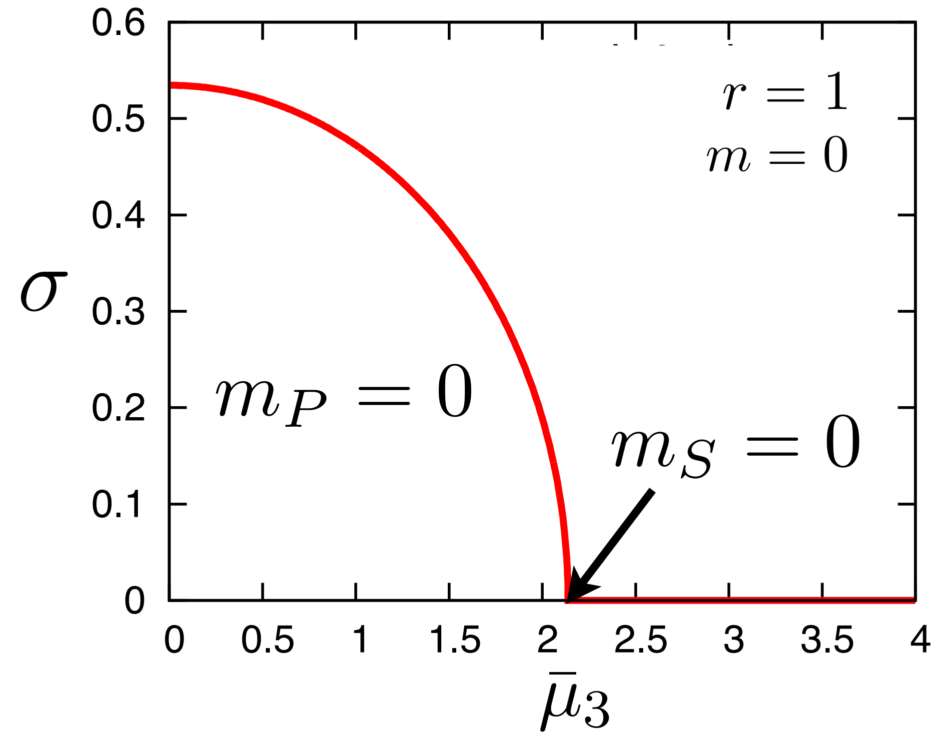


- *m=0*

  <u>2nd-order phase transition</u>

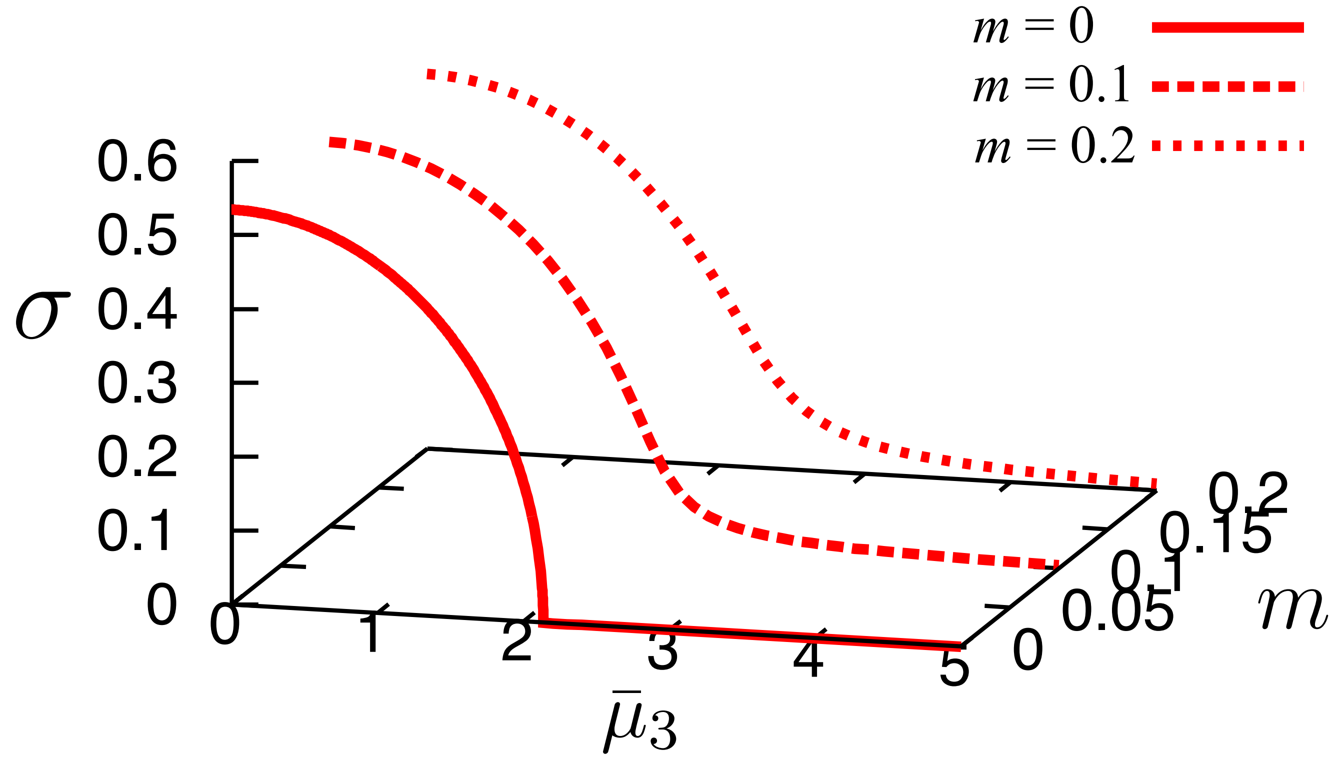
  Divergent correlation length on the boundary

  → m_P=0 m_S=0



- *m≠0*

  <u>2nd-order → crossover</u>



ii) 2d Gross-Neveu model (large N)

$$S = \frac{1}{2}\sum_{n,\nu}\bar{\psi}_n\gamma_\nu(\psi_{n+\nu}-\psi_{n-\nu}) + \frac{1}{2}\sum_n \bar{\psi}_n i\gamma_2(2\psi_n - \psi_{n+\hat{1}} - \psi_{n-\hat{1}})$$
$$-\frac{1}{2N}\sum_n \left[g_\sigma^2(\bar{\psi}_n\psi_n)^2 + g_2^2(\bar{\psi}_n i\gamma_2\psi_n)^2\right] + \mu\sum_n \bar{\psi}_n i\gamma_2\psi_n$$

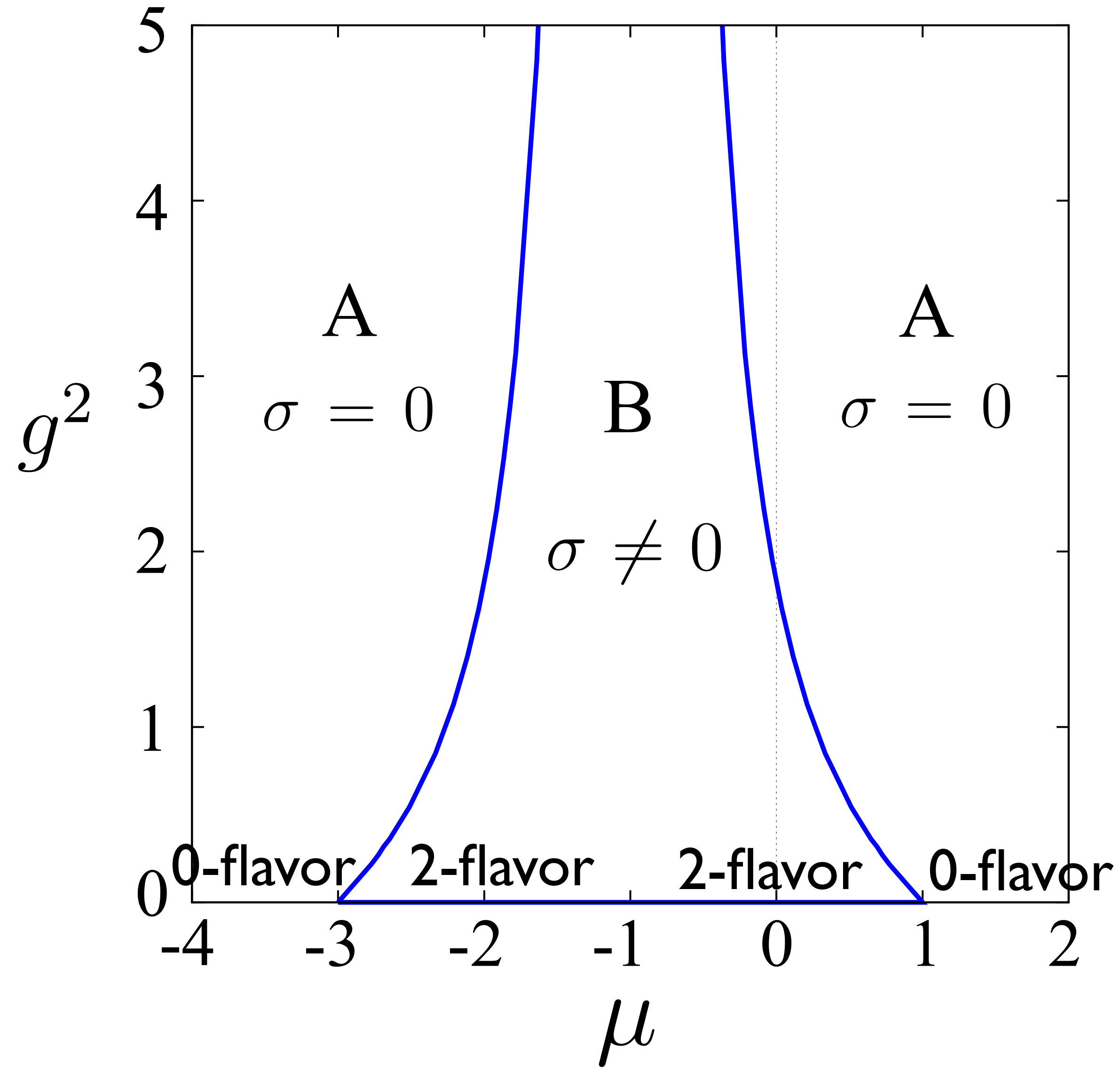
- Phase boundary

  2-flavor & 0-flavor boundary

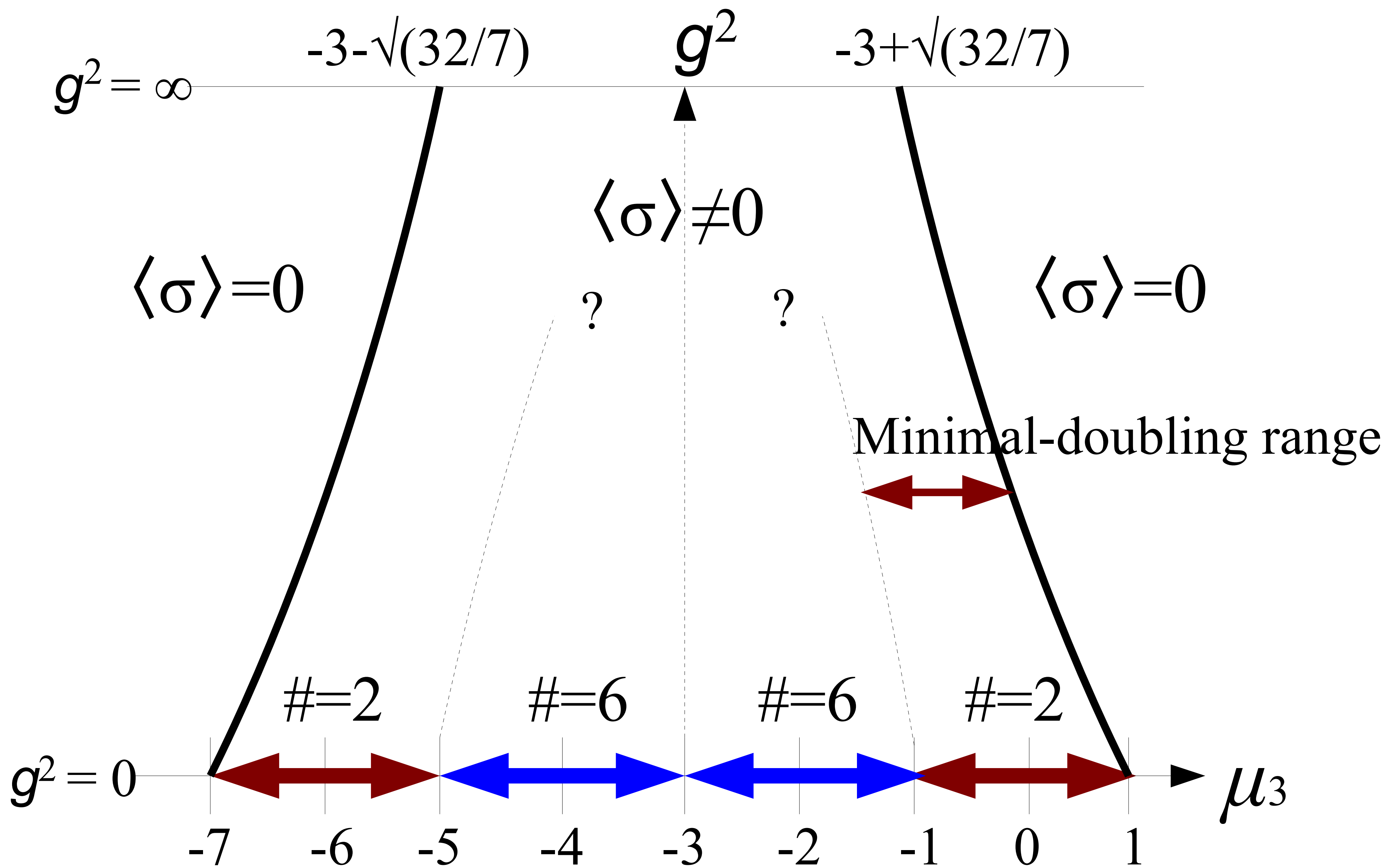
  → *σ=0 means no fermion d.o.f*

- Minimal-doubling range

  narrower in strong coupling



Conjecture on 4d lattice QCD with MD



μ_3 should be set in MD range.

4. (T-μ) phase diagram

i) Strong-coupling limit

1. Link variable integral
2. Bosonization → meson potential
3. Determine the vacuum

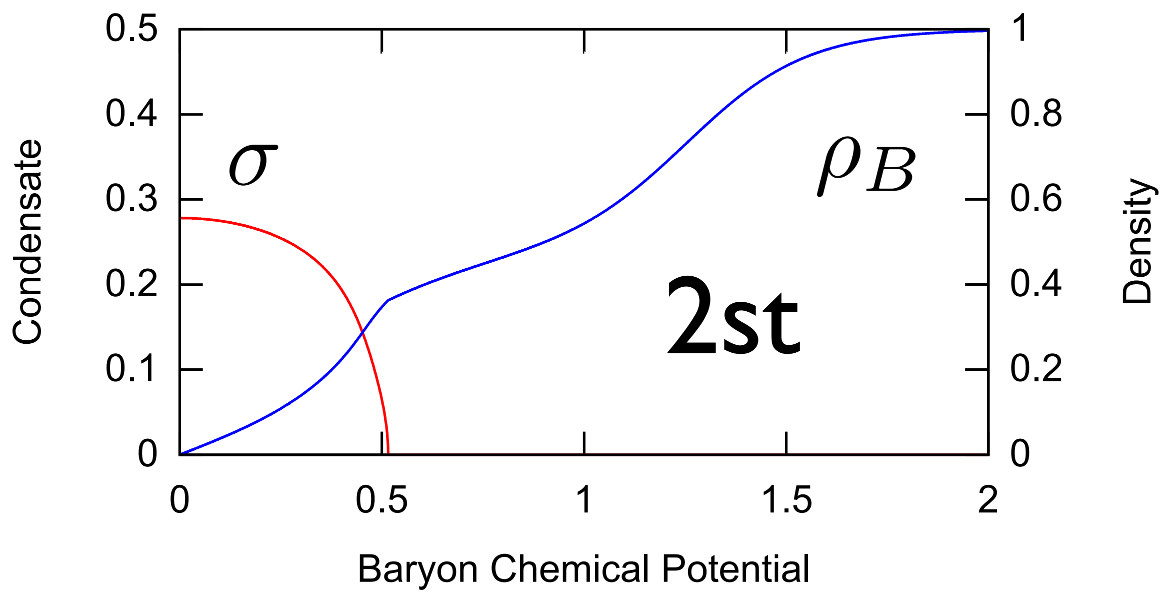
→ Finite-(T,μ) case [Fukushima-Hatsuda-Nishida '04]

- Meson effective potential

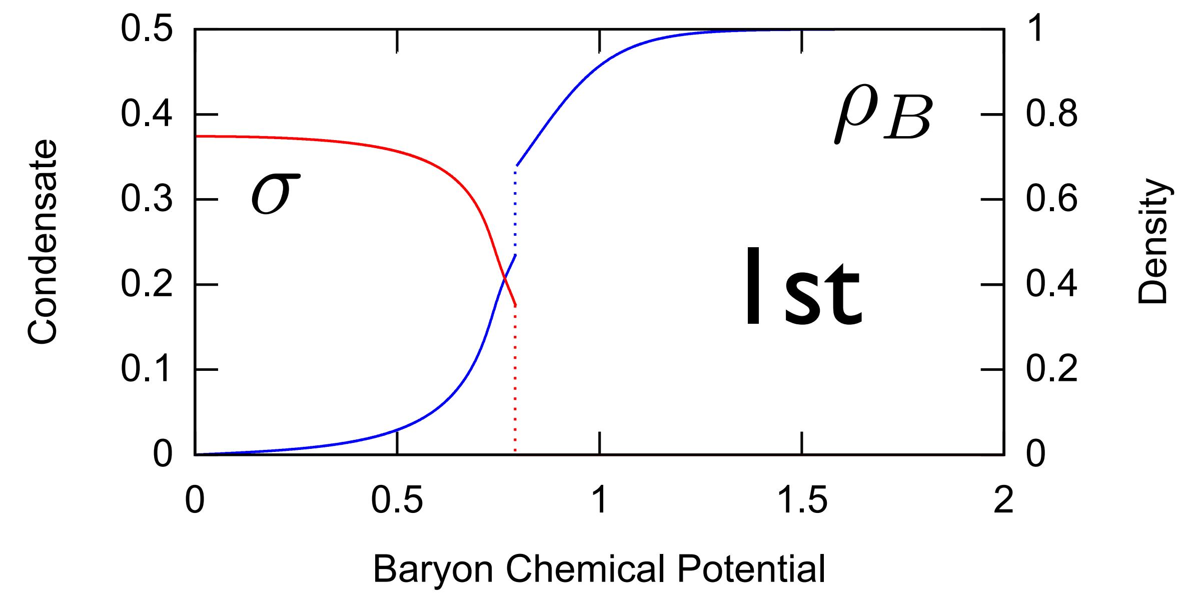
$$\mathcal{F}_{\rm eff}(\sigma,\pi_4;m,T,\mu,\mu_3) = \frac{N_c D}{4}\left((1+r^2)\sigma^2+(1-r^2)\pi_4^2\right) - N_c\log A$$
$$-\frac{T}{4}\log\left(\sum_{n\in\mathbb{Z}}\det\left(Q_{n+i-j}\right)_{1\le i,j\le N_c}\right).$$

Chiral condensate & Baryon density

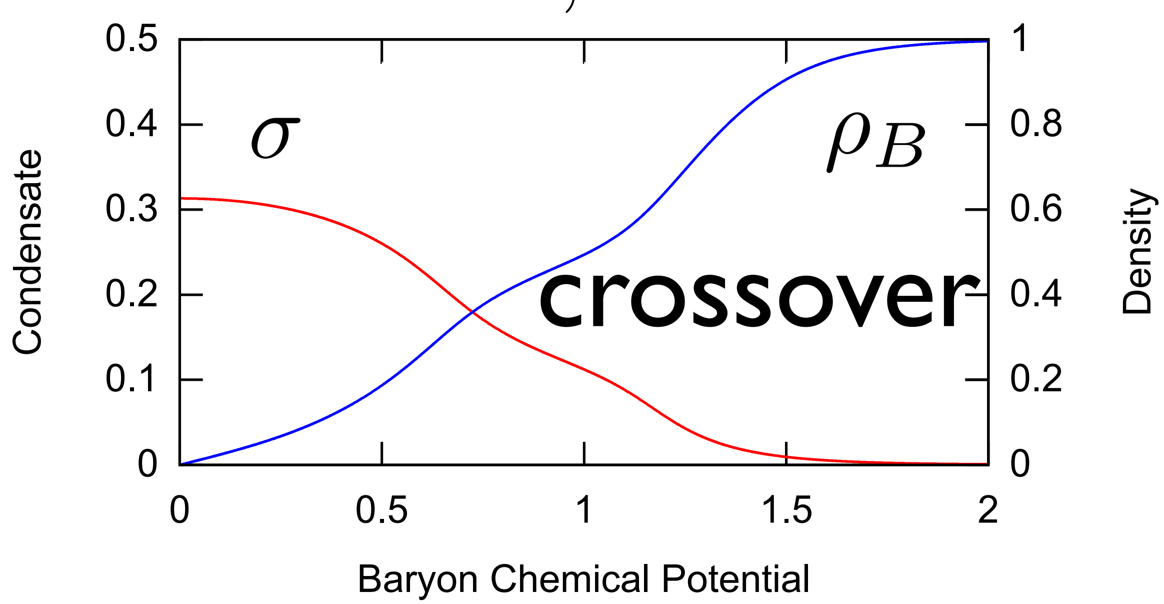
$m = 0,\ T = 0.3$



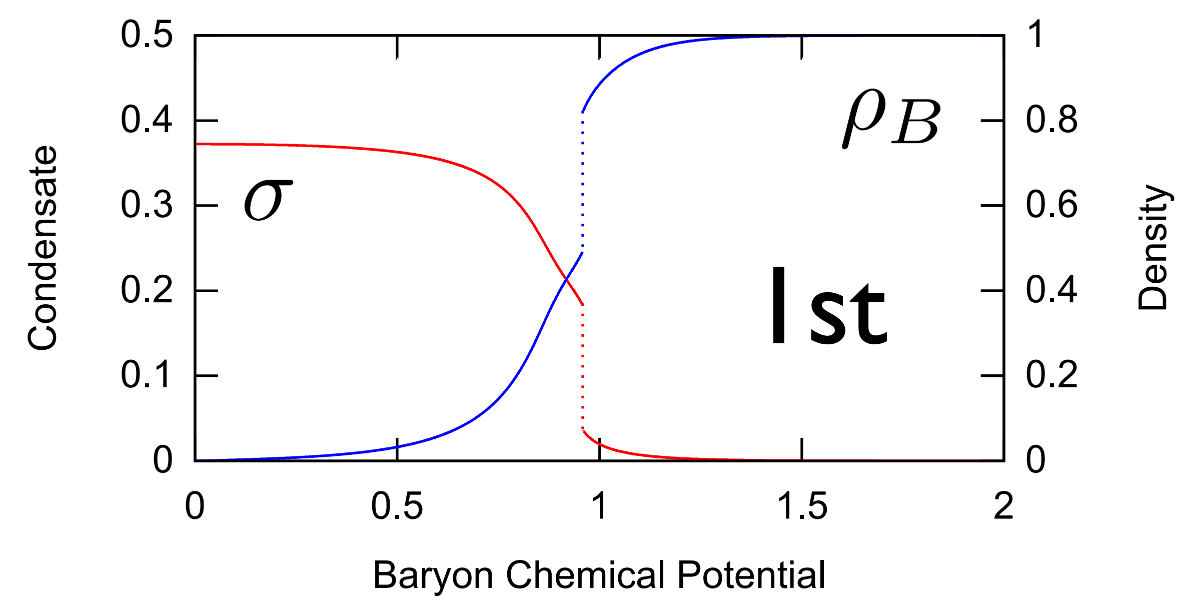
$m = 0,\ T = 0.2$



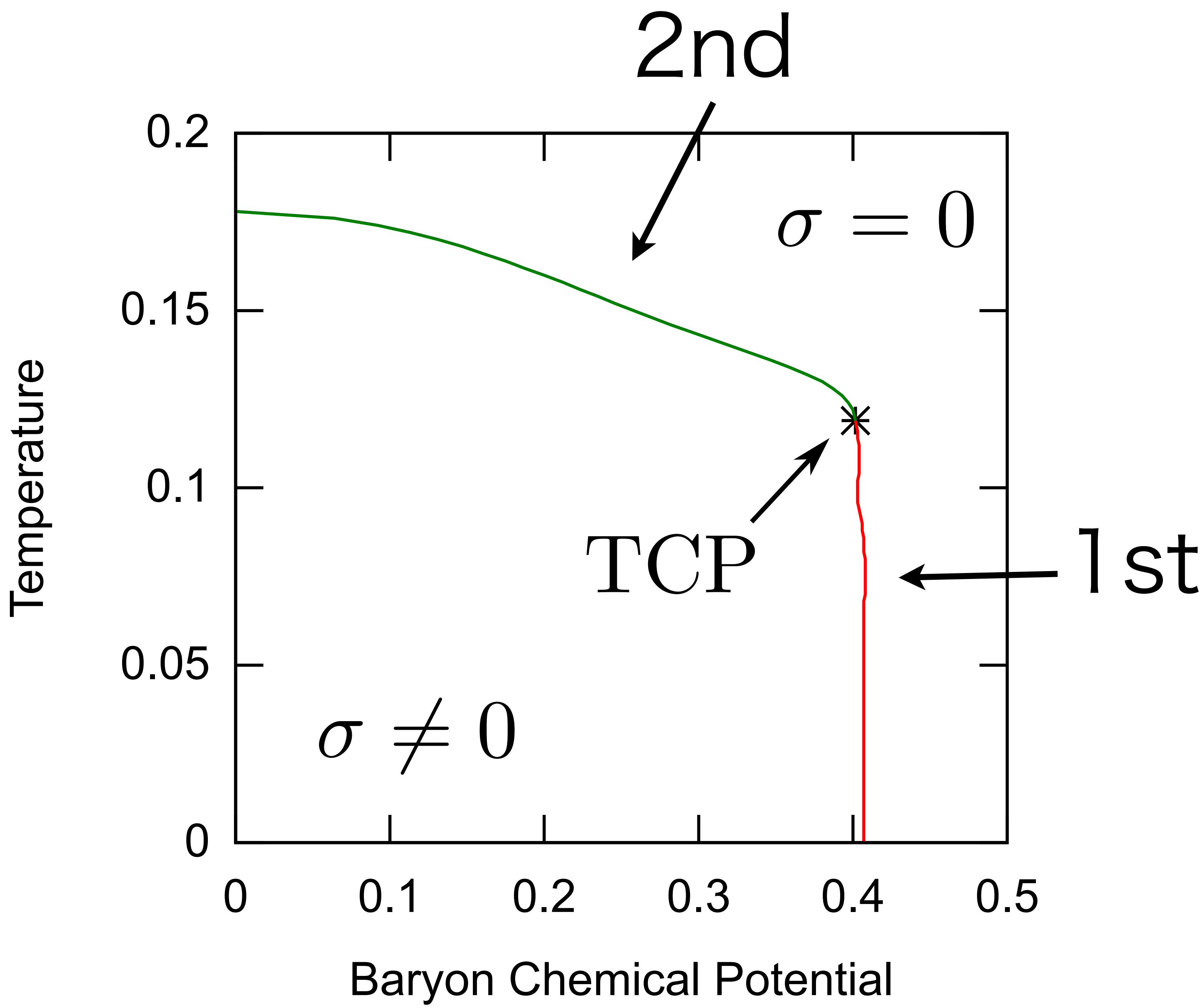
$m = 0.1,\ T = 0.3$



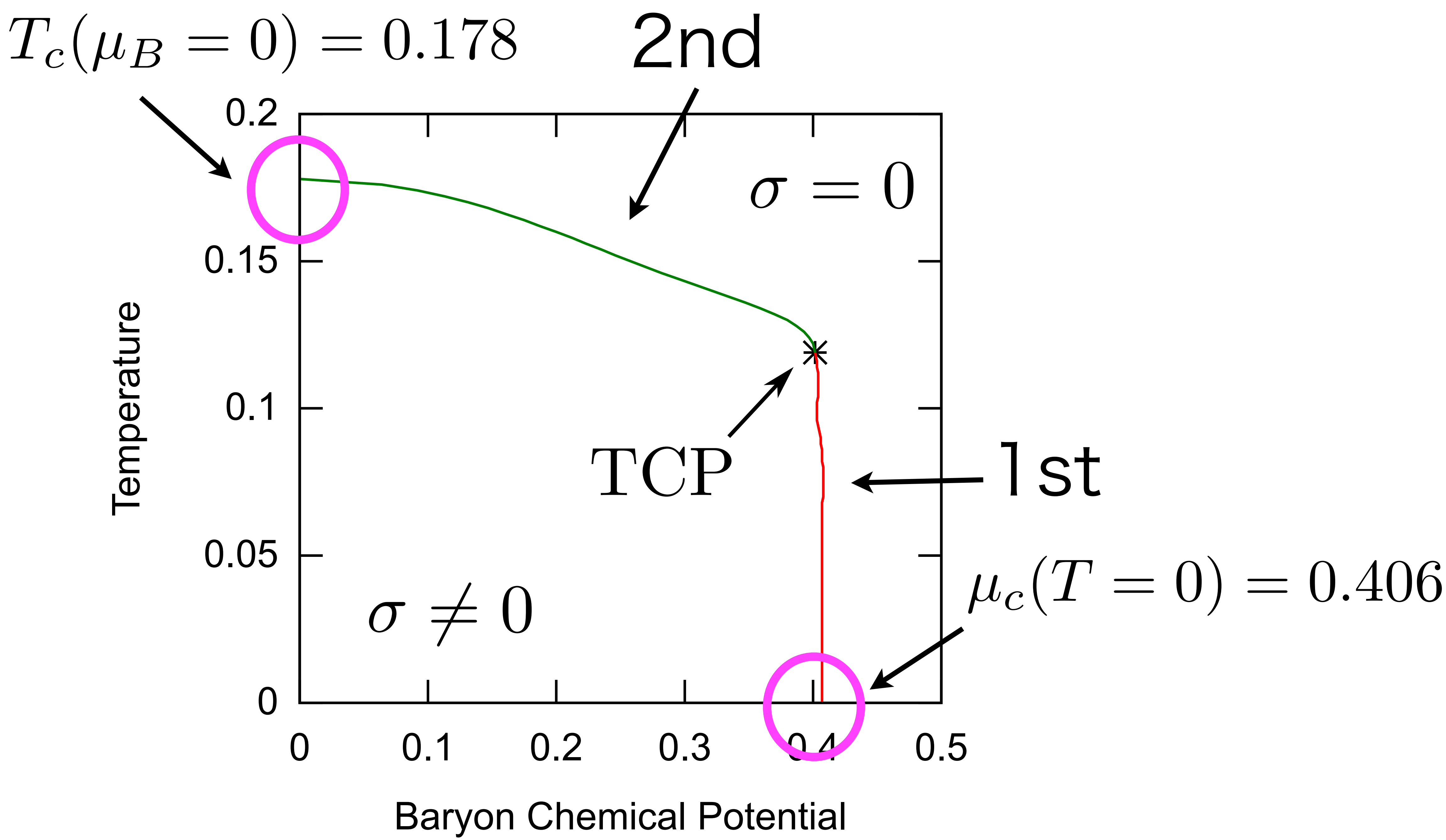
$m = 0.1,\ T = 0.2$



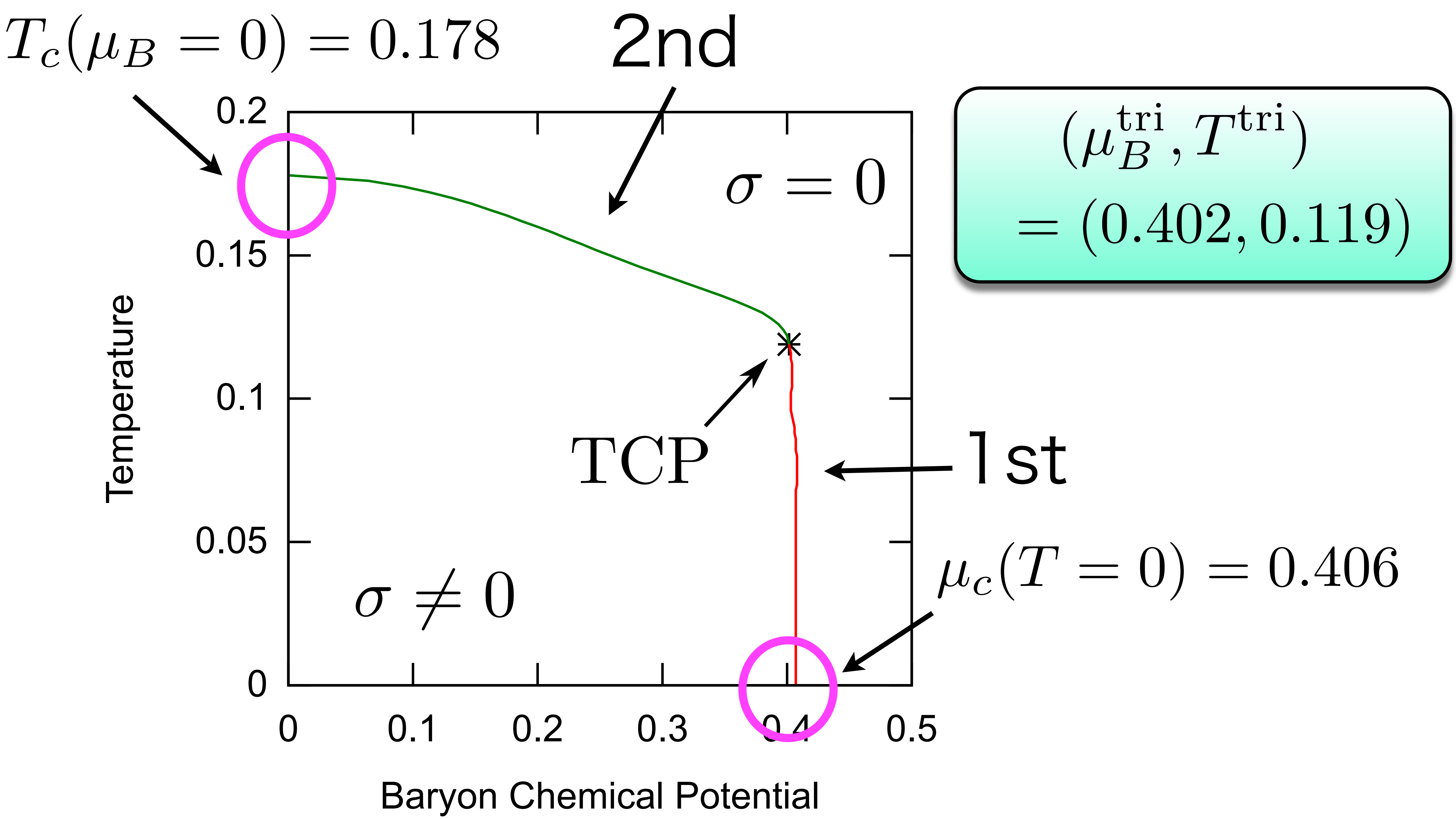
Phase diagram



Phase diagram



Phase diagram



Phase diagram

- Critical density/temp ratio
  - KW fermion : $R^0_{\mathrm{KW}} = \dfrac{\mu_c(T=0)}{T_c(\mu_B=0)} \sim 2.3$
  - Staggered : $R^0_{\mathrm{st}} \sim 1$
  - Phenomenology : $R^0_{\mathrm{ph}} \gtrsim 5.5$

Phase diagram

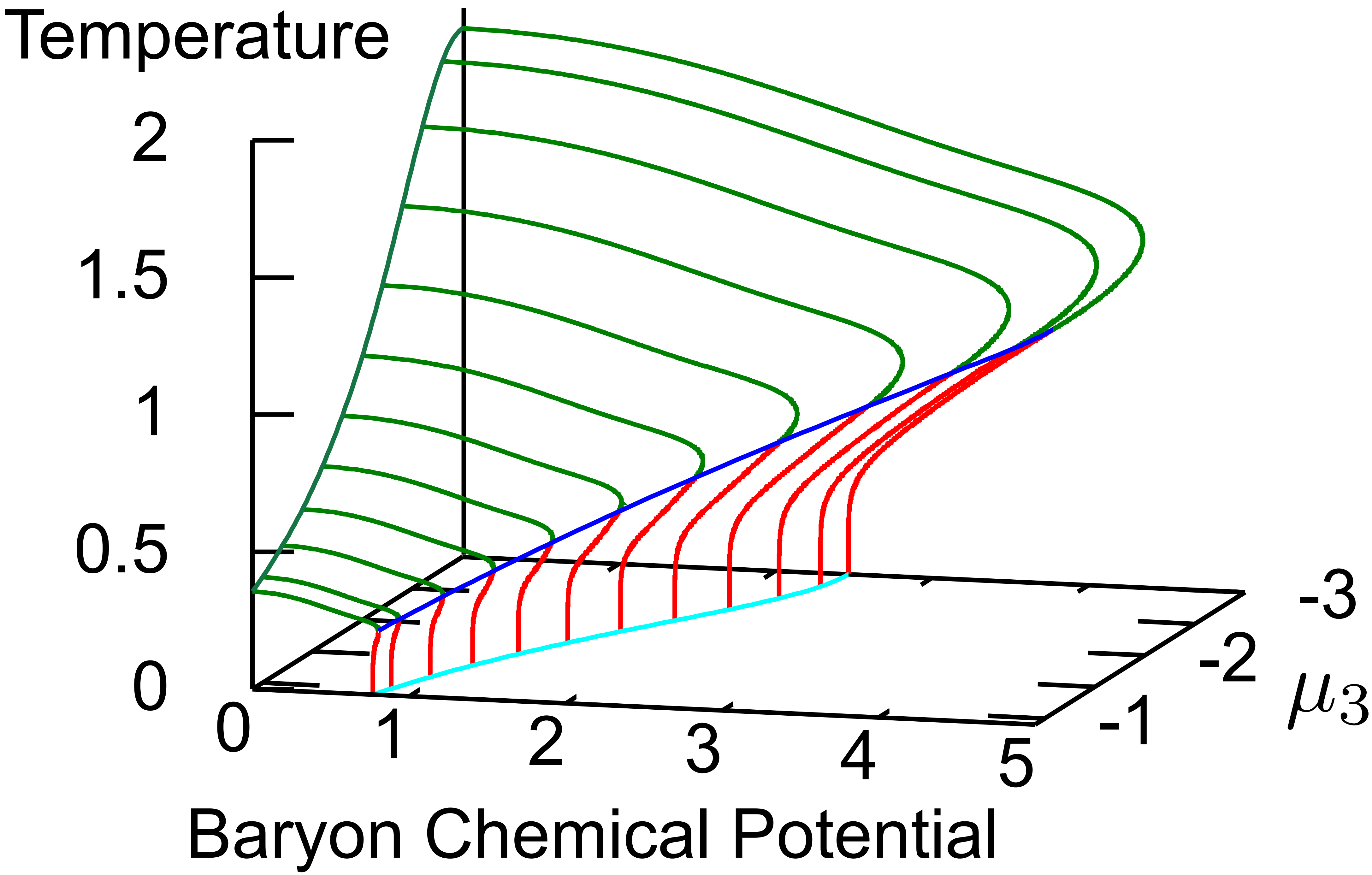
- Tricritical point ratio
  - KW fermion : $R_{\mathrm{KW}}^{\mathrm{tri}} = \dfrac{\mu_B^{\mathrm{tri}}}{T^{\mathrm{tri}}} \simeq 3.4$
  - Staggered : $R_{\mathrm{st}}^{\mathrm{tri}} \simeq 2.0$
  - Monte-Calro simulation : $R_{\mathrm{MC}}^{\mathrm{tri}} \gtrsim 3$

Summary

- Flavored chemical potential is another way of reducing species doublers.
- The symmetries of the formulation imply it suits finite temperature and density system.
- We find chiral phase structure in parameter spaces.
- (T,μ) chiral phase diagram is close to phenomenological conjectures.

Future works

- In this talk we concentrate on Imaginary flavored μ + O(1) real μ.
- We can also study Real flavored μ with fine-tuning of μ_3.


Temperature
2
1.5
1
0.5
0
0
1
2
3
4
5
Baryon Chemical Potential
-3
-2
-1
μ_3

Effective potential

$$\mathcal{F}_{\mathrm{eff}}(\sigma,\pi_4;m,T,\mu,\mu_3) = \frac{N_c D}{4}\left((1+r^2)\sigma^2+(1-r^2)\pi_4^2\right) - N_c \log A - \frac{T}{4}\log\left(\sum_{n\in\mathbb{Z}} \det\left(Q_{n+i-j}\right)_{1\le i,j\le N_c}\right).$$

for Nc = 3

$$\begin{aligned}
&\sum_{n\in\mathbb{Z}} \det\left(Q_{n+i-j}\right)_{1\le i,j\le N_c} \\
&= 8\left(1+12\cosh^2\frac{E}{T}+8\cosh^4\frac{E}{T}\right)\left(15-60\cosh^2\frac{E}{T}+160\cosh^4\frac{E}{T}-32\cosh^6\frac{E}{T}+64\cosh^8\frac{E}{T}\right) \\
&+64\cosh\frac{\mu_B}{T}\cosh\frac{E}{T}\left(-15+40\cosh^2\frac{E}{T}+96\cosh^4\frac{E}{T}+320\cosh^8\frac{E}{T}\right) \\
&+80\cosh\frac{2\mu_B}{T}\left(1+6\cosh^2\frac{E}{T}+24\cosh^4\frac{E}{T}+80\cosh^6\frac{E}{T}\right) \\
&+80\cosh\frac{3\mu_B}{T}\cosh\frac{E}{T}\left(-1+\cosh^2\frac{E}{T}\right)+2\cosh\frac{4\mu_B}{T}, \qquad (19)
\end{aligned}$$

Effective potential

with

$$A^2 = 1 + \left(\mu_3 + Dr - \frac{D}{2}\sqrt{1-r^2}\pi_4\right)^2, \qquad B = m + \frac{D}{2}\sqrt{1+r^2}\sigma,$$

$$E = \operatorname{arcsinh}\left(\frac{B}{A}\right) = \log\left[\frac{B}{A} + \sqrt{1 + \left(\frac{B}{A}\right)^2}\right],$$

Dispersion relation

$$
\begin{aligned}
S_{\mathrm{KW}} = \sum_n \Big[& \frac{1}{2} \sum_{\mu=1}^{4} \bar{\psi}_n \gamma_\mu (U_{n,n+\mu}\psi_{n+\mu} - U_{n,n-\mu}\psi_{n-\mu}) \\
& + \frac{r}{2} \sum_{j=1}^{3} \bar{\psi}_n i\gamma_4 (2\psi_n - U_{n,n+j}\psi_{n+j} - U_{n,n-j}\psi_{n-j}) + \mu_3 \bar{\psi}_n i\gamma_4 \psi_n + m\bar{\psi}_n\psi_n \\
& + \frac{d_4}{2} \bar{\psi}_x \gamma_4 (U_{n,n+4}\psi_{n+4} - U_{n,n-4}\psi_{n-4}) \Big] ,
\end{aligned}
\tag{2.8}
$$

$$
D(p) \sim i\gamma_i p_i + i\gamma_4 p_4 \sqrt{(1+d_4)^2 - \mu_3^2} + O(ap^2) \, . \qquad \textbf{(free)}
$$

$$
V(p,k) = -ig_0\Big(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 \sqrt{(1+d_4)^2 - \mu_3^2}\Big) + O(ap, ak) \, .
$$